



SATIPS

Research
Education
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MATHEMATICS

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Editorial

Welcome to the Autumn 2019 maths broadsheet. In this term's broadsheet, I share a problem I use on the first day of the new school year to set a mathematical tone for our year. I also present a research article about the importance of varying the context of word problems (and even expert mathematicians struggle with some of the word problems!). Finally, I mention two small books that could become valuable resources for teachers new to mathematics or to advanced pupils looking to learn even more.

As you move through the term, please consider sharing with us your lesson ideas, your thoughts about mathematics, and your questions, particularly ways to provide our pupils with positive mathematical experiences. Please send them to the address above for inclusion in a future broadsheet.

A Problem to Start off the Year

I'm sure I have shared this problem before, but it is one I revisit myself at the start of each new school year because of its ability to draw pupils into some of the interesting problem-solving aspects of mathematics. It is also accessible to pupils of a wide range of ages and abilities.

Each year, when I meet my pupils for the first time, I like to begin with some maths. (Many of my colleagues begin with rules and expectations first. These are certainly necessary aspects of a new school year, and they find that approach works well for them.) To set a mathematical tone for the year, I present this problem I first learned about from Douglas Black at <https://mathematicscentre.com/taskcentre/030truth.htm>:

I ask pupils what they notice and what they wonder about this image. Generally, pupils mention the nine empty boxes, the operation signs (add, subtract, and multiply), the equals signs, and that there are three equations. Some pupils predict they will need to put numbers into the boxes. We discuss their ideas and I tell them they need to use the numbers 1 to 9 to fill the empty boxes. Since there are nine numbers (1, 2, 3, 4, 5, 6, 7, 8, and 9) and nine empty boxes, they need to put one number per box, using all nine numbers and without using any number more than once. They need to make sure that all three equations are true at the same time. If they need an example, I fill in numbers like this:

$$\square + \square = \square$$

$$\square - \square = \square$$

$$\square \times \square = \square$$

$$1 + 2 = 3$$

$$4 - 5 = 6$$

$$7 \times 8 = 9$$



Little Quick Fix series by John MacInnes (Sage Publishing): **Know Your Numbers** and **Understanding Probability**.

Sage Publishing has released a new series of small mathematics guidebooks. Designed for those conducting research, several of these books seem rather appropriate for both teachers new to teaching mathematics as well as advanced pupils. Each of these books is pocket-sized (for those with moderately-large pockets), not extremely long (roughly 100 to 150 pages or so), and is written in clear, concise language. Each book is organised into several sections and includes checkpoints and questions for readers to be certain of their understanding before moving on.

The **Know Your Numbers** book covers fractions, decimals, and percentages, as well as organizing numbers into basic tables, rounding, and ratios. **Understanding Probability** is a nice introduction to that topic, beginning with “What is probability and why does it matter?” and moving through calculating probability and conditional probability. The final section goes slightly past what might be expected of your pupils, but not too far (advanced pupils will enjoy the challenge!).

This series contains nearly twenty different books, each covering a different topic related to academic research. Not every title is appropriate for this level (for example, “Find the Theory in Your Research” is likely to be of less interest to you and your pupils at this stage), but the two titles discussed here do have direct applications to our mathematics at this level. Prices online seem to hover in the area of £7 to £8, making them a reasonably priced resource to have in the staff room library.



Pupils look at the example and I ask them what they notice. They might say they see I have used all 9 numbers (none more than once), that there is one number per box, that the first equation is true ($1 + 2 = 3$) but that the other two equations are not true. This example is usually enough to start them thinking and problem-solving.

I let pupils work with a partner (or individually if they prefer) and mini-whiteboards are perfect for this activity. One reason I like this activity is that it gives pupils an opportunity to interact with each other and discuss the problem. Not all of my pupils already know one another on the first day of school, so talking with new people about a maths problem gives a focused opportunity to interact (quite important for more introverted pupils).

This is also a problem in which a correct solution is unlikely to appear the first time pupils insert numbers into boxes. It helps reinforce the idea that mistakes are a part of mathematics. Sometimes, pupils' incorrect placement of numbers can help lead them to a correct solution. Allowing pupils some time for productive struggle is an important part of this problem-solving process.

If pupils need a hint (but don't provide it too early!), I might refer to the third equation (the multiplication one). I might ask which numbers cannot go in that equation. Pupils might tell me that 1 is not allowed there because any number times 1 gives the number we started with (such as $8 \times 1 = 8$) and we can't use a number more than once. A pupil might also notice that if 1 is not allowed

in that equation, then a number larger than 4 can't be a factor (for example $2 \times 5 = 10$, and 10 is not a number we can use for this problem). Note that my hint is phrased as a series of questions designed to help pupils think about the problem, rather than “Put a 3 in this box.”

Once pupils find one solution, I challenge them to find another solution. At this stage, some pupils use the commutative property and rearrange an equation (for example they might change $2 \times 3 = 6$ into $3 \times 2 = 6$). Other pupils may swap their addition and subtraction equations by rewriting them ($7 + 1 = 8$ might become $8 - 7 = 1$). These are quite valuable explorations for pupils as they investigate properties of numbers.

Initially, some people think this problem is far too simple (it is, after all, just three quite simple equations). The fact that pupils are limited to the numbers 1 to 9 (one time each) and the three equations have to be true at the same time provides the challenge. And for the first day of school, it is a worthwhile challenge that helps set a mathematical tone for our year together!

Trouble with Word Problems? Your Pupils Are Not Alone.

If your pupils are anything like mine, they find word problems (sometimes called story problems) challenging, even when they already know how to solve the mathematics involved. It turns out, pupils are not the only ones who find word problems challenging. Even expert mathematicians can struggle with the same type of word problems our pupils struggle with.

Three researchers in France recently published a paper titled “When masters of abstraction run into a concrete wall: Experts failing arithmetic word problems.” When given a word problem that can be solved using the equation $14 - 2 = 12$, the adult problem-solvers were highly influenced by the context of the problem. For example, consider these two problems (both problems are taken from the article, referenced below):

Joe takes a Russian dictionary weighing 5 kg. He also takes a Spanish dictionary. In total, he is carrying 14 kg of books. Lola takes Joe's Spanish dictionary and a German dictionary. The German dictionary weighs 2 kg less than the Russian dictionary. How many kilograms of books is Lola carrying?

Tom took painting classes for 5 years. He started taking painting classes at a specific age. He stopped taking the classes at the age of 14 years. Lucy started taking painting classes at the same age as Tom. She took classes for 2 years less than him. How old was Lucy when she stopped taking painting classes?

The mathematical structure of the two problems is identical, but the context is different. The first problem is considered *cardinal* (problems which involve collections, prices, and weights/masses) and the second problem is *ordinal* (those problems involving duration, distance, or floor numbering).

The researchers discovered that the context of the problem (the wording, not the mathematical structure) influenced the way the adults represented and solved the word problems. (Their thought process was much different depending on whether the problem was cardinal or ordinal.) The adults were nearly twice as likely to solve the ordinal problems correctly than to solve the cardinal problems correctly.

These results seem to support the idea that we need to be presenting our pupils with a wide range of word problems, addressing a wide range of situations. We also need to be modelling visual ways of solving these problems and encouraging our pupils to make and share their visual representations. By seeing a range of these representations, pupils will have more opportunity to become flexible in their mathematical thinking, especially when presented with problems in new contexts.

The entire article is available online at <https://archive-ouverte.unige.ch/unige:120760/ATTACHMENT01> or by searching for the article's title on Google Scholar.



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Courses and Events

A selection of forthcoming courses for Autumn 2019:

Venue : London

- 14 Oct Strategies for using iPads/ tablets in class teaching
- 14 Oct Online E-safety for schools and classrooms
- 16 Oct Art Scholarship
- 18 Oct Improving Pupil Progress by metacognition & self-regulation
- 4 Nov Moving on to Middle Management
- 8 Nov Gender Differences in the Classroom
- 11 Nov Holistic teaching in the EYFS
- 11 Nov EYFS Teaching with Inspiration
- 15 Nov Lesson Observation and Performance Management
- 22 Nov Leading Music to Outstanding
- 27 Nov Art Scholarship
- 18 Nov KS2 English Conference
Venue: The King's School Canterbury Kent

For more information on this terms Courses please see the SATIPS website or contact the SATIPS Course Director.

These courses will run as training days in London, Bristol, Birmingham or York.

The cost of the day courses includes follow-up project based work and one to one feedback. They are also available as inset days.

Bespoke training packages for schools are available with discount for more than one course booked.

For more information please email the team on training@satips.org or telephone **07584 862263**.

Prep School Magazine

'Prep School' is published three times a year. It offers readers in prep schools a broad range of authoritative articles on educational issues.

Editor: Paul Jackson (eajackson22@hotmail.com)



Competitions, Exhibitions and Events for pupils

SATIPS offers a variety of pupil-focused events. Over many years schools have enjoyed entering their pupils in events that have a nationwide attraction with high standards. These events include:

- **SATIPS Challenge** (annual general knowledge quiz)
- **National Handwriting Competition**
- **Poetry Competition**
- **SATIPSKI**
- **Annual Art Exhibition**

Full details of all these events are at www.satips.org/competitions